Analyzing the Quantum Theory of the Scattering of X-Rays by Light Elements and Its Follow-Up

Part I: On the Quantum Theory of the Scattering of X-Rays by Light Elements

Introduction and Organization of the Paper

The 1923 article by Arthur Holly Compton titled A Quantum Theory of the Scattering of X-Rays by Light Elements is one of the most remarkable achievements of the field of physics in the first part of the twentieth century. It provided both experimental and theoretical information that light in particular the X-rays can act as particles according to the quantum theory. The article is organized logically and composed like early physical studies - a mixture of tentative empirical data collection and a theoretical account on new conjectures about quantum. Compton broke his work into distinct parts that have logical connections with background theory, experimental design, derivation of mathematical expressions and lastly interpretation of results. He begins by explaining some earlier experiments of X-ray scattering and the inadequacy of the classical wave theory to give an explanation as to why the observed wavelength variations happened. At this point he introduces the idea of X-rays being a series of discrete quanta (photons) which hit the electrons like the balls in a billiard game. This framework anticipates additional mathematical and experiment analysis, which leads to the development of what is now known as Compton wavelength shift equation. The form of the paper indicates the way Compton attempted to steer the readers through what classical electromagnetic theory failed to accomplish to the need to have a quantum mechanical explanation.

Main Results and Theoretical Contributions

Concerning its major findings, the paper by Compton offers a theoretical prediction and experimental confirmation of the wavelength change that is achieved when X-rays are scattered by light elements like carbon, beryllium, and paraffin. The fundamental theoretical finding is

the Compton equation of the wavelength shift given by $\Delta\lambda = \frac{h}{mc}(1-\cos\theta)$,

where h is the Planck constant, m is the rest mass of the electron, c is the speed of light and 0 is the scattering angle. This term is used to measure the dependence on the wavelength of scattered X-rays (that is, the energy loss of the photon) with respect to the angle in which the photon is deflected. To arrive at this relation, Compton used the conservation of energy and momentum when acting on the collision of a photon with a freely moving electron at rest. The simplicity and the direct correlation between the formula and the quantities that are measurable made it a great tool to test photon model of light. In practice, Compton determined the X-ray scattering at different angles with the help of crystal spectrometers and found that the secondary X-ray lines have longer wavelengths than the one of the incident radiation. These measurements were only slightly off his theoretical forecasts, which proved that the scattered photons actually were losing a measurable amount of energy which was equal to the recoil of the electrons. Such a twofold confirmation, mathematical and empirical, played a vital role in

persuading the physics community that light could act as particles, which possess discrete quantities of energy and momentum.

Conclusions and Scientific Implications

Compton did not only reach conclusions that were limited to the X-ray scattering phenomenon. His work was one of the earliest definite illustrations of the duality of light in that although it is known to be wave-like, e.g. that it can interfere and diffract, it also acts as a particle stream when it interacts with individual atoms or electrons. His model was successful, which gave great authority to his previous hypothesis of the photoelectric effect that had been proposed by Einstein (1905), which was also based on the quantization of the light energy. In the final parts of the paper Compton indicated that the classical theory of electromagnetism, in which light is a continuous wave, could not explain the discrete exchange of energy which can be observed in the scattering process. He observed that it was only through the assumption of photons

with even momentum $p=\frac{h}{\lambda}$ that one could be able to match theory with data observed. Also, the amount of the shift, he found, was independent of the material employed, and only depended on the angle of scattering,--another important point of evidence in support of his quantum model. Therefore, the findings of Compton proved the reality of photon momentum and also provided a new and quantifiable constant of nature

the Compton wavelength of the electron, $\frac{h}{mc} \approx 2.43 \times 10^{-12}$ meters. Not only was this finding able to substantiate the validity of quantum mechanics, but it also paved the way to advances in the field of quantum electrodynamics.

Hypotheses and Assumptions

The assumptions and the hypothesis that underlie the analysis used by Compton are an essential aspect that can be used to comprehend the rationale behind his conclusions. Some of them were made directly and some were implicit in the manner he developed his model. The most obvious hypothesis is that X-rays may be treated as discrete quanta of energy $E=h\nu$ which has linear momentum $p=\frac{h}{L}$. This was an assumption, not developed on first principle in the paper, but rather imported out of the accruing body of quantum theory, created by Planck and Einstein. Another assumption made by Compton specifically was that the scattering mechanism would be treated as an elastic collision between a photon and an electron, where both the energy and momentum were conserved. It was this simplification that enabled him to consider the electron as a free particle although in practice electrons are confined in atoms. He explained this by the idea that in cases where the energy of the X-rays was sufficiently high, the binding energies of the electrons in light elements were very small in comparison with the photon energy.

Some of the unspoken assumptions included Compton assuming that the electrons that were being scattered were originally at rest, that relativistic effects were calculable using the energy-momentum relations by Einstein and that the scattering process happened in isolation (each electron separately) without considering the collective atomic effects. Also, he assumed that the detector had enough resolution and calibration on the small wavelength shifts to measure them in a reliable manner which was later found to be true in experiments.

Final Evaluation and Historical Significance

The 1923 article by Compton is not just a landmark in the chronicle of physics, however, it is also a paradigm of scientific thought in the sense that it balances theory and experiment upon definite presuppositions and impeccable measurements. The paper is organized in such a way that the logic of discovery is manifested: a problem is formulated (the unexplained behavior of scattered X-rays), a theoretical model is proposed (collisions of photons with electrons), a quantitative prediction is made and tested. This reserved face and the uniformity of the findings with the experimental data testify to the fact that Compton realises the implications of his discoveries in general. His work gave the earliest definitive evidence that electromagnetic radiation has momentum, which had been argued over since Maxwell, and changed the description of light-matter interactions. In addition, the paper had an impact on the new area of quantum mechanics, which motivated

subsequent research on the duality in particle and wave behavior as well as the quantum field theory. Although some of his simplifying assumptions would be improved on later, especially the treatment of bound electrons and relativistic corrections, the fundamental point still stands today. The mixture of theoretical understanding and strict experimentation by Compton is a classic example of the empirical ethos of the early quantum physics, and a turning point in the establishment of the photon concept. Overall, his 1923 article is a clear and strong example of how phenomena beyond the reach of classical physics might be attributed to the quantum nature of light, which represented a definite leap in the development of the modern science.

Part II: On the Transfer in Quanta of Radiation Momentum to Matter

Introduction

In 1923, William Duane wrote an article, The Transfer in Quanta of Radiation Momentum to Matter, published in the Proceedings of the National Academy of Sciences, about the exchange of radiation and matter in discrete portions or quanta. Duane tried to give an explanation of the reflection and diffraction of X-rays by the crystals using quantum instead of classical wave theory. His work was based on previous experimental findings with G. L. Clark who demonstrated the X-rays reflected off crystals in a manner not entirely explicable by interference. According to Duane, a new framework radiation momentum, just like energy, is a quantized quantity, and energy and momentum are conserved in the process of interaction with matter. The paper is well structured so it is presented in a way that it moves through theoretical assumptions to mathematical formulation and then to its application in optical phenomena, which are the strength of the quantum approach.

Theoretical Background and Structure.

Duane starts by questioning the adequacy of classical theory of wave interference when trying to do explanations about crystal reflections. Instead, he suggests that radiation would act on matter by discrete momentum exchanges controlled by the constant of Blanchet (h)

of Planck. This is the core postulation of his theory: the transfer of momentum is proportional to h/L of radiation and matter in quanta where L is the characteristic length like the space between crystal planes. With this concept, Duane develops the X-ray reflection in a simple crystal in two planes that are orthogonal and simplified. The X-ray quantum with energy hv and momentum hv/c impinges incidentally on the crystal, imparting momentum in discrete pieces along the axes of the crystal. Dimensional reasoning allows Duane to write these components of

momentum as $M_x=\frac{rh}{a}$ and $M_y=\frac{r'h}{b}$ with r and r' integers. Replacing them with conservation equations he derives equations that give just Bragg Law ($n\lambda=2\,d\sin\theta$) without any reference to wave interference an impressive achievement that relates crystal reflection directly to quantum momentum exchange.

Main Results and Equations

The core of the paper by Duane is that he derives the equations which characterize the diffraction and reflection completely as a result of quantized transfer of momentum. His two-dimensional case is generalized to three dimensions, with crystal parameters $d_1, d_2, \wedge d_3$, and with integer quantum numbers $r_1, r_2, \wedge r_3$. The equations that come out are the following,

$$\left(1-l'\right)\frac{e}{c}=r_1\frac{h}{d_1}, \left(m-m'\right)\frac{e}{c}=\frac{r_2 h}{d_2}, \left(n-n'\right)\frac{e}{c}=\frac{r_3 h}{d_3},$$

act as overall quantum constraints to X-ray diffraction. Duane demonstrates that under these conditions the Law of Bragg is derived, and these directions provide the directions of characteristic X-ray reflections of elements in a crystal. He then applies the theory to the observable phenomena of light, namely, reflection, refraction, and diffraction, and shows that the same quantization rules lead to the rederival of the Laws of Snell and of the thin-film interferences. Even Duane extends the principle to the photoelectric effect, giving an alternative version of the energy equation, that incorporates the idea of a momentum transfer to the ejected electron and the atom recoiled, refining slightly the original formula of Einstein. His analysis, throughout, shows that quantum mechanics can explain a whole host of optical phenomena that were originally due to waves exclusively.

Hypotheses and Assumptions

The work of Duane relies on implicit and explicit assumptions. He explicitly states that radiation momentum ($\frac{hv}{c}$) is carried off in integral multiples of h divided by some appropriate measure of the structure and that the law of conservation of energy and momentum are always correct. He directly ignores the recoil energy of the crystal as the crystal is very massive and thus it makes calculations easier. Implicitly, he presupposes that the X-rays and light may be discussed as discrete particles instead of being considered continuous waves, which is a major

twist to the classical theory. He further assumes that quantization of momentum is a universal phenomenon of electromagnetic radiation, not limited to X-rays but also to the visible light. Lastly, the dimensional reasoning adopted by Duane assumes that the proportionality constant between the transfer of momentum and h could be selected to be unity; this was done because of the simplicity of it and not out of any first-principles argument. Though these assumptions are not concrete, they allow one to make equations that are in close correspondence with the experimental data, which proves the practical utility of his model.

Conclusion

In making the shift to quantum models of light, the paper by Duane in 1923 was an important step towards replacing the wave-based models. The structure of the paper, the hypothesis and its application are organized in a logical way which introduces a coherent pattern of comprehending the exchange of radiation and matter on a unitary level. His derivations were able to replicate the Law of Bragg and classical optical relations indicating that wave phenomena are possible to emerge naturally due to quantum interactions. Although Duane acknowledges that his line of reasoning is not a logical demonstration, his method provides a strong heuristic that is based on quantization and conservation. The paper concludes by stating that every physical theory, which is constructed on basic assumptions is aimed to primarily guide experiment. Reflectively, the fact that radiation momentum is quantized

was an insightful contribution by Duane, which quelled future discoveries in quantum electrodynamics and enhanced the comprehension of the wave-particle duality of light. His work is still considered to be a fundamental contribution towards early quantum physics and study of X-ray and optical interactions.

Reference:

Compton, A. H. (1923). A quantum theory of the scattering of X-rays by light elements. *Physical review*, *21*(5), 483.

Dune, William (1923). The transfer in quanta of radiation momentum to matter. *Proceedings of the National Academy of Sciences*, *9*(5), 158-164.